OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS **4731**

Mechanics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 19 June 2009
Afternoon

Duration: 1 hour 30 minutes

* 4 7 3 1 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

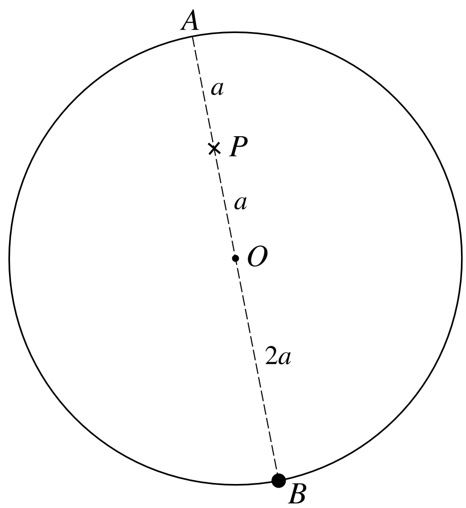
1 A top is set spinning with initial angular speed $83\,\text{rad}\,\text{s}^{-1}$, and it slows down with constant angular deceleration. When it has turned through 1000 radians, its angular speed is $67\,\text{rad}\,\text{s}^{-1}$.

(i) Find the angular deceleration of the top. **[2]**

(ii) Find the time taken, from the start, for the top to turn through 400 radians. **[4]**

2 The region R is bounded by the x-axis, the lines $x = a$ and $x = 2a$, and the curve $y = \frac{a^3}{x^2}$ for $a \leqslant x \leqslant 2a$, where a is a positive constant. A uniform solid of revolution is formed by rotating R through 2π radians about the x-axis. Find the x-coordinate of the centre of mass of this solid. **[7]**

3



A uniform circular disc has mass $4m$, radius $2a$ and centre O. The points A and B are at opposite ends of a diameter of the disc, and the mid-point of OA is P. A particle of mass m is attached to the disc at B. The resulting compound pendulum is in a vertical plane and is free to rotate about a fixed horizontal axis passing through P and perpendicular to the disc (see diagram). The pendulum makes small oscillations.

(i) Find the moment of inertia of the pendulum about the axis. **[4]**

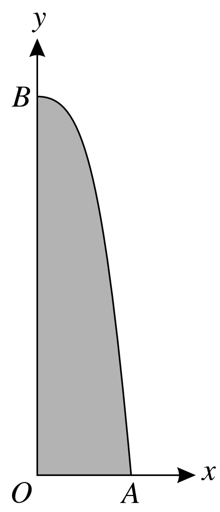
(ii) Find the approximate period of the small oscillations. **[4]**

4 From a helicopter, a small plane is spotted 3750 m away on a bearing of 075°. The plane is at the same altitude as the helicopter, and is flying with constant speed $62\,\text{m}\,\text{s}^{-1}$ in a horizontal straight line on a bearing of 295°. The helicopter flies with constant speed $48\,\text{m}\,\text{s}^{-1}$ in a straight line, and intercepts the plane.

(i) Find the bearings of the two possible directions in which the helicopter could fly. **[5]**

(ii) Given that interception occurs in the shorter of the two possible times, find the time taken to make the interception. **[4]**

5



A uniform lamina of mass 63 kg occupies the region bounded by the x-axis, the y-axis, and the curve $y = 8 - x^3$ for $0 \leqslant x \leqslant 2$. The unit of length is the metre. The vertices of the lamina are O (0, 0), A (2, 0) and B (0, 8) (see diagram).

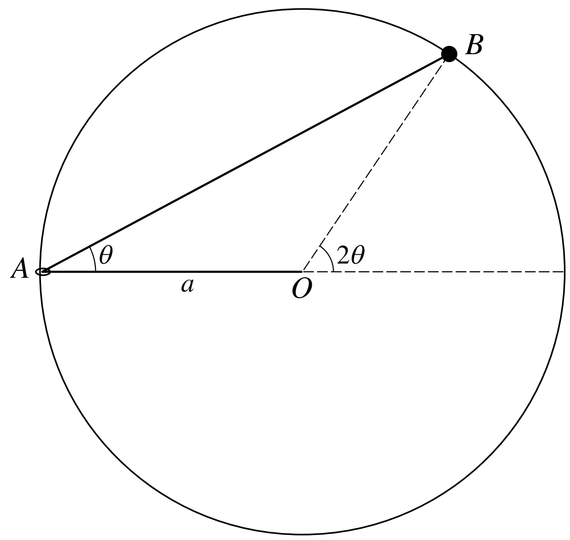
(i) Show that the moment of inertia of this lamina about OB is 56 kg m^2. **[6]**

It is given that the moment of inertia of the lamina about OA is 1036.8 kg m^2, and the centre of mass of the lamina has coordinates $\left(\frac{4}{5}, \frac{24}{7}\right)$. The lamina is free to rotate in a vertical plane about a fixed horizontal axis passing through O and perpendicular to the lamina. Starting with the lamina at rest with B vertically above O, a couple of constant anticlockwise moment 800 N m is applied to the lamina.

(ii) Show that the lamina begins to rotate anticlockwise. **[2]**

(iii) Find the angular speed of the lamina at the instant when OB first becomes horizontal. **[6]**

6



A smooth circular wire, with centre O and radius a, is fixed in a vertical plane, and the point A is on the wire at the same horizontal level as O. A small bead B of mass m can move freely on the wire. A light elastic string, with natural length a and modulus of elasticity $\sqrt{3}mg$, passes through a fixed ring at A, and has one end fixed at O and the other end attached to B. The section AB of the string is at an angle θ above the horizontal, where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$, so that OB is at an angle 2θ to the horizontal (see diagram).

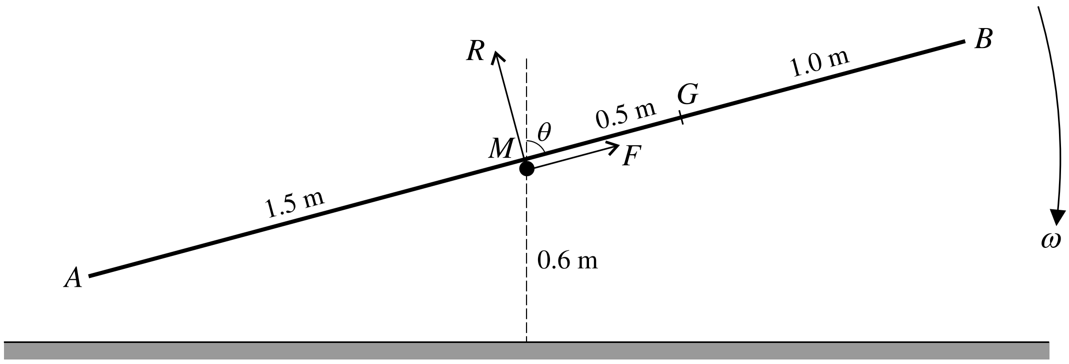
(i) Taking O as the reference level for gravitational potential energy, show that the total potential energy of the system is

$$mga(\sqrt{3} + \sqrt{3}\cos 2\theta + \sin 2\theta).$$ **[4]**

(ii) Find the two values of θ for which the system is in equilibrium. **[5]**

(iii) For each position of equilibrium, determine whether it is stable or unstable. **[4]**

7



A thin horizontal rail is fixed at a height of 0.6 m above horizontal ground. A non-uniform straight rod AB has mass 6 kg and length 3 m; its centre of mass G is 2 m from A and 1 m from B, and its moment of inertia about a perpendicular axis through its mid-point M is $4.9\,\text{kg}\,\text{m}^2$. The rod is placed in a vertical plane perpendicular to the rail, with A on the ground and M in contact with the rail. It is released from rest in this position, and begins to rotate about M, without slipping on the rail. When the angle between AB and the upward vertical is θ radians, the rod has angular speed $\omega\,\text{rad}\,\text{s}^{-1}$, the frictional force in the direction AB is F N, and the normal reaction is R N (see diagram).

(i) Show that $\omega^2 = 4.8 - 12\cos\theta$. **[3]**

(ii) Find the angular acceleration of the rod in terms of θ. **[2]**

(iii) Show that $F = 94.8\cos\theta - 14.4$, and find R in terms of θ. **[6]**

(iv) Given that the coefficient of friction between the rod and the rail is 0.9, show that the rod will slip on the rail before B hits the ground. **[4]**

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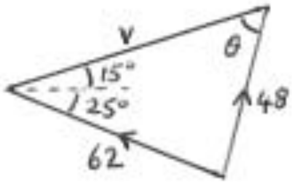
1 (i)	Using $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $67^2 = 83^2 + 2\alpha \times 1000$ $\alpha = -1.2$ Angular deceleration is 1.2 rad s^{-2}	M1 A1 [2]	
(ii)	Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $400 = 83t - 0.6t^2$ $t = 5 \text{ or } 133\frac{1}{3}$ Time taken is 5 s	M1 A1ft M1 A1 [4]	 Solving to obtain a value of t
	Alternative for (ii) $\omega_2^2 = 83^2 - 2 \times 1.2 \times 400$ M1A1 ft $\omega_2 = 77$ $77 = 83 - 1.2t$ M1 $t = 5$ A1		*(M0 if $\omega = 67$ is used in (ii))*

2	Volume $V = \int \pi y^2 \mathrm{d}x = \int_a^{2a} \pi \frac{a^6}{x^4} \mathrm{d}x$	M1	*π may be omitted throughout*
	$= \pi \left[-\frac{a^6}{3x^3} \right]_a^{2a} = \frac{7}{24}\pi a^3$	A1	For integrating x^{-4} to obtain $-\frac{1}{3}x^{-3}$
	$V\bar{x} = \int \pi x y^2 \mathrm{d}x$	M1	for $\int xy^2 \mathrm{d}x$
	$= \int_a^{2a} \pi \frac{a^6}{x^3} \mathrm{d}x$	A1	Correct integral form (including limits)
	$= \pi \left[-\frac{a^6}{2x^2} \right]_a^{2a} = \frac{3}{8}\pi a^4$	A1	For integrating x^{-3} to obtain $-\frac{1}{2}x^{-2}$
	$\bar{x} = \frac{\frac{3}{8}\pi a^4}{\frac{7}{24}\pi a^3}$	M1	*Dependent on previous M1M1*
	$= \frac{9a}{7}$	A1 [7]	

3 (i)	$I = \frac{1}{2}(4m)(2a)^2 + (4m)a^2$ $+ m(3a)^2$ $= 21ma^2$	M1 A1 B1 A1 [**4**]	Applying parallel axes rule
(ii)	From P, $\bar{x} = \dfrac{(4m)a + m(3a)}{5m}$ $\left(= \dfrac{7a}{5} \right)$ Period is $2\pi\sqrt{\dfrac{21ma^2}{5mg(\frac{7}{5}a)}}$ $= 2\pi\sqrt{\dfrac{3a}{g}}$	M1 M1 A1 ft A1 [**4**]	Correct formula $2\pi\sqrt{\dfrac{I}{mgh}}$ seen *or* using $L = I\ddot{\theta}$ and period $2\pi / \omega$
	Alternative for (ii) $-4mga\sin\theta - mg(3a)\sin\theta = (21ma^2)\ddot{\theta}$ M1 M1 Period is $2\pi\sqrt{\dfrac{21ma^2}{7mga}} = 2\pi\sqrt{\dfrac{3a}{g}}$ A1 ft A1		Using $L = I\ddot{\theta}$ with three terms Using period $2\pi / \omega$

4 (i)	 $\frac{\sin\theta}{62}=\frac{\sin 40}{48}$ $\theta = 56.1°$ *or* $123.9°$ Bearings are $018.9°$ and $311.1°$	M1 M1 A1 A1A1 [5]	Velocity triangle One value sufficient Accept $19°$ and $311°$
(ii)	Shorter time when $\theta = 56.1°$ $\frac{v}{\sin 83.87}=\frac{48}{\sin 40}$ Relative speed is $v = 74.25$ Time to intercept is $\frac{3750}{74.25}$ $= 50.5$ s	B1 ft M1 M1 A1 [4]	Or $v^2 = 62^2 + 48^2 - 2\times 62\times 48\cos 83.87$ *Dependent on previous M1*
	Alternative for (i) and (ii) $\begin{pmatrix}48\sin\phi\\48\cos\phi\end{pmatrix}t=\begin{pmatrix}3750\sin 75\\3750\cos 75\end{pmatrix}+\begin{pmatrix}62\sin 295\\62\cos 295\end{pmatrix}t$ M1 M1 $3.732\cos\phi - \sin\phi = 3.208$ A1 M1 M1 $\phi = 18.9°$ and $311.1°$ A1A1 B1 ft $t = 50.5$ A1		component eqns (displacement or velocity) obtaining eqn in ϕ *or* t *or* v $(=3750/t)$ correct simplified equation or $t^2 - 231.3t + 9131.5 = 0$ $[t = 50.5,\ 180.8]$ or $v^2 - 94.99v + 1540 = 0$ $[v = 74.25,\ 20.74]$ solving to obtain a value of ϕ solving to obtain a value of t *(max A1 if any extra values given)* appropriate selection for shorter time

5 (i)	Area is $\int_0^2 (8-x^3)\,dx = \left[\, 8x - \tfrac{1}{4}x^4 \,\right]_0^2 = 12$ Mass per m^2 is $\rho = \frac{63}{12} = 5.25$ $I_y = \sum(\rho\, y\, \delta x)x^2 = \rho \int x^2 y\,dx$ $= \rho \int_0^2 (8x^2 - x^5)\,dx$ $= \rho \left[\, \tfrac{8}{3}x^3 - \tfrac{1}{6}x^6 \,\right]_0^2 = \frac{32}{3}\rho$ $= \frac{32}{3} \times \frac{63}{12} = 56\ \text{kg m}^2$	B1 M1 M1 A1 A1 A1 AG [**6**]	 for $\int x^2 y\,dx$ or $\int x^3\,dy$ or $\tfrac{1}{3}\rho \int_0^8 (8-y)\,dy$ for $\frac{32}{3}$
(ii)	Anticlockwise moment is $800 - 63 \times 9.8 \times \tfrac{4}{5}$ $= 306.08\ \text{N m} \ > 0$ so it will rotate anticlockwise	M1 A1 [**2**]	Full explanation is required; (anti)clockwise should be mentioned before the conclusion
(iii)	$I = I_x + I_y = 1036.8 + 56 \quad (= 1092.8)$ WD by couple is $800 \times \tfrac{1}{2}\pi$ Change in PE is $63 \times 9.8 \times \left(\tfrac{24}{7} - \tfrac{4}{5}\right)$ $800 \times \tfrac{1}{2}\pi = \tfrac{1}{2}I\omega^2 - 63 \times 9.8 \times \left(\tfrac{24}{7} - \tfrac{4}{5}\right)$ $1256.04 = 546.4\omega^2 - 1622.88$ $\omega = 2.30\ \text{rad s}^{-1}$	B1 B1 B1 M1 A1 A1 [**6**]	 Equation involving WD, KE and PE *May have an incorrect value for I; other terms and signs are cao*

6 (i)	GPE is $mg(a\sin 2\theta)$ $\text{AB} = 2a\cos\theta$ *or* $\text{AB}^2 = a^2 + a^2 - 2a^2\cos(\pi - 2\theta)$ EPE is $\frac{\sqrt{3}mg}{2a}(2a\cos\theta)^2$ $= \sqrt{3}mga(1+\cos 2\theta)$ Total PE is $V = \sqrt{3}mga(1+\cos 2\theta) + mga\sin 2\theta$ $= mga(\sqrt{3} + \sqrt{3}\cos 2\theta + \sin 2\theta)$	B1 B1 M1 A1 AG [**4**]	Or $mg(2a\cos\theta\sin\theta)$ Any correct form Expressing EPE and GPE in terms of $\cos 2\theta$ and $\sin 2\theta$
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga(-2\sqrt{3}\sin 2\theta + 2\cos 2\theta)$ $= 0$ when $2\sqrt{3}\sin 2\theta = 2\cos 2\theta$ $\tan 2\theta = \frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{12},\ -\frac{5\pi}{12}$	B1 M1 M1 A1A1 [**5**]	(B0 for $\frac{\mathrm{d}V}{\mathrm{d}\theta} = -2\sqrt{3}\sin 2\theta + 2\cos 2\theta$) Solving to obtain a value of θ Accept $0.262,\ -1.31$ or $15°,\ -75°$
(iii)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga(-4\sqrt{3}\cos 2\theta - 4\sin 2\theta)$ When $\theta = \frac{\pi}{12}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -8mga < 0$ so this position is unstable When $\theta = -\frac{5\pi}{12}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 8mga > 0$ so this position is stable	B1ft M1 A1 A1 [**4**]	 Determining the sign of V'' *or* M2 for alternative method for max / min

7 (i)	Initially $\cos\theta = \frac{0.6}{1.5} = 0.4$ $\frac{1}{2}\times 4.9\omega^2 = 6\times 9.8(0.5\times 0.4 - 0.5\cos\theta)$ $\omega^2 = 12(0.4-\cos\theta)$ $\omega^2 = 4.8 - 12\cos\theta$	M1 A1 A1 AG [**3**]	Equation involving KE and PE
(ii)	$6\times 9.8\times 0.5\sin\theta = 4.9\alpha$ $\alpha = 6\sin\theta \quad (\text{rad s}^{-2})$	M1 A1 [**2**]	or $2\omega\frac{d\omega}{d\theta} = 12\sin\theta$ or $2\omega\frac{d\omega}{dt} = 12\sin\theta\frac{d\theta}{dt}$
(iii)	$6\times 9.8\cos\theta - F = 6\times 0.5\omega^2$ $58.8\cos\theta - F = 14.4 - 36\cos\theta$ $F = 94.8\cos\theta - 14.4$ $6\times 9.8\sin\theta - R = 6\times 0.5\alpha$ $58.8\sin\theta - R = 18\sin\theta$ $R = 40.8\sin\theta$	M1 M1 A1 AG M1 M1 A1 [**6**]	for radial acceleration $r\omega^2$ radial equation of motion *Dependent on previous M1* for transverse acceleration $r\alpha$ transverse equation of motion *Dependent on previous M1*
(iv)	If *B* reaches the ground, $\cos\theta = -0.4$ $F = -52.32$ $\sin\theta = \sqrt{0.84}$ [$\theta = 1.982$ *or* $113.6°$] $R = 37.39$ Since $\frac{52.32}{37.39} = 1.40 > 0.9$, this is not possible	M1 A1 M1 A1 [**4**]	*Allow M1A0 if* $\cos\theta = +0.4$ *is used* Obtaining a value for *R* Or $\mu R = 33.65$, and $52.32 > 33.65$
	Alternative for (iv) Slips when $F = -0.9R$ $94.8\cos\theta - 14.4 = -36.72\sin\theta$ M1 $\theta = 1.798$ [$103.0°$] A1 *B* reaches the ground when $\cos\theta = -0.4$ M1 $\theta = 1.982$ [$113.6°$] so it slips before this A1		 *Allow M1A0 if* $F = +0.9R$ *is used* *Allow M1A0 if* $\cos\theta = +0.4$ *is used*